MONTHLY WEATHER REVIEW

Editor, ALFRED J. HENRY

VOL. 56, No. 12
W. B. No. 974

DECEMBER, 1928

CLOSED FEBRUARY 2, 1929
ISSUED MARCH 2, 1929

A CRITIQUE ON THE CONSTRUCTION AND USE OF MINIMUM-TEMPERATURE FORMULAS

By ECKLEY S. ELLISON

[Weather Bureau, Pomona, Calif., October 21, 1928]

INTRODUCTION

Considerable attention during the last decade has been directed toward the development of methods for forecasting the minimum temperature using empirical mathematical formulas. Many formulas have been suggested, some of which are inherently faulty. This paper attempts to consider the subject as an entity to ascertain the relative merits of the different systems proposed, and to delineate the limitations that necessarily must exist in the construction and application of minimum temperature formulas in actual forecasting practice.

GENERAL CONSIDERATIONS

Frost usually occurs under conditions of high barometric pressure or when the weather is passing to anticyclonic control following the progression of a cyclone. Atmospheric stability is pronounced. Weather changes are due chiefly to insolation and rising temperature in the daytime coupled with free radiation and falling temperature at night. From the regularity of the nocturnal fall in temperature under ideal frost conditions, i. e., clear skies, no wind to disturb the stratification of the lower layers of the atmosphere, and the temperature falling under the influence of free radiation to a point below freezing at sunrise, it is but a step to the enumeration and mathematical expression of the various factors that operate collectively to produce these changes.

Under ideal frost conditions the more important factors that influence the minimum temperature at any given point can be enumerated:

1. *Temperature of the radiating surface.*—The rate of radiation varies as the 4th power of the absolute temperature of the radiating surface. Let this factor be expressed symbolically as T.

2. *Length of night, or interval during which free radiation takes place.*—Symbolically, t.

3. *Absolute humidity.*—The atmosphere would be practically diathermanous were it not for the presence in it of water vapor, which possesses the property of absorbing to some extent the outgoing waves of heat radiation. The rate of radiation is inversely proportional to the absolute humidity. Symbolically expressed as A.

4. *Liberation of latent heat from condensation and fusion of atmospheric moisture under special conditions.*—Symbolically expessed as L.

5. *Topographical situation of station.*—Points on the valley floor, in general, are colder than points on the surrounding hillsides due to the effects of air drainage. (1) The minimum temperature at any point, then, would depend upon the degree of temperature inversion prevailing and the relative location of this point in the stratified lower atmosphere. Symbolically expressed as a function of the temperature inversion, $f\ (I)$.

If we denote the minimum temperature by y, the general equation for its evaluation under ideal frost conditions becomes:

$$y = f(T) + f(t) + f(A) + f(L) + f(I) \qquad (1)$$

It is reasonable to assume that the factor $f(t)$ can be neglected provided the data are developed by frost seasons, since the average length of the night would remain practically constant over the period of extreme frost danger in the short spring and autumn frost periods, while the longer winter frost period is centered over the winter solstice when the nights become progressively longer to the solstice and then become progressively shorter at the same rate.

INDEX TO TEMPERATURE OF RADIATING SURFACE

Humphreys states (2): "* * * the temperature of the surface layer of the atmosphere is chiefly controlled by the temperature of the greedily absorbing and freely radiating surface of the earth." If a thermometer is exposed 4½ feet above the ground in a fruit region (4) or other standard thermometer shelter, it has been found by experiment that even where the surface texture of the ground is made to vary between the extremes of bare black soil and luxuriant vegetation, there is very littler difference in temperature that can be attributed to the condition of the ground or radiating surface. (5) (6) (7). For the purpose of this paper it can be inferred that the free air temperature at a point 4½ feet above the ground is a direct function of the temperature of the freely absorbing and radiating ground surface, and as such can be used in lieu of the temperature of the radiating surface.

Now when it is considered that the rate of radiation at any given instant during the night chiefly depends upon the pair of values to be associated with the temperature of the radiating surface and the absolute humidity, and that for every pair of values so associated there is a corresponding pair of values for the temperature of the dew point and the relative humidity, it is possible to express equation 1, empirically, as

$$y = f(d) + f(h) + f(I) \quad (2)$$

where d and h are the temperature of the dew point and the relative humidity, respectively.

The factor $f(L)$ is considered to be taken care of by the new factor $f(h)$, since the higher the numerical value of the relative humidity at sunset, the greater is the probability of the dew point temperature being reached, and conversely. Some error necessarily is introduced here since the factor $f(L)$ is only indirectly represented by the factor $f(h)$, but when it is considered that the air temperature often remains quite stationary for several hours after the temperature of the dew point has been reached, it is seen that this error usually is of small order. This function, now discarded, will be considered again later.

Let us now leave equation 2 in its present form and proceed to the development of the argument which ultimately will lead to the evaluation of the factor $f(I)$.

INFLUENCE OF EXPOSURE

Temperature inversion is a phenomenon met with wherever frost occurs. It is due to the fact that the thermal conductivity of air is so poor that superstrata of different temperature may exist. At night, under ideal frost conditions, the ground surface is cooled by the free radiation of its heat below the temperature of the air immediately adjacent. This surface layer of air becomes chilled through the conduction of its heat to the colder ground surface, while its increased density holds it to the ground; in fact, even is responsible for its motion or drift to the surface point of lowest elevation.

Other conditions being the same, the amount of temperature inversion will be greater following the day with the higher maximum temperature. Also will the degree of temperature inversion be greater on those nights when the values of dew point and relative humidity are relatively low, for it is on such nights that the rate of effective radiation approaches its maximum and minimum temperatures are lowest.

Recent studies in temperature inversion have brought out the fact that very large differences in temperature often exist between hill and valley stations (**8**) (**9**) (**10**) (**11**), or at different elevations on towers erected in cold orchards (**12**). Young (**9**) has found a site in southern California where the minimum temperature at a hillside station less than one-half mile distant and 225 feet higher than a base station was on the average nearly 17° F. higher than at the base station, with extreme differences as great as 28° F. and as small as 8° F.

Let us assume that temperature formulas were to be constructed for each of these stations, using equation 2. It is evident that the absolute humidity at both stations would be subject only to minute local variations and that the only practical difference in the formulas would be in the values assigned to the factor $f(I)$. The adiabatic temperature difference is here neglected owing to its doubtful existence and its small theoretical value. For the base station, $f(I) = 0$, and would vanish. For the hill station, $f(I) = +17°$, with extreme errors of application ranging from $-9°$ to $+11°$, or more than sufficient to nullify any accuracy to be obtained by computation with the other factors.

In every air drainage system there is some one point where the slowly moving air first gathers to form the nucleus of the pool of cold air that later covers the valley lowlands. Such a point is called a key point and the temperature station at the key point is called a key station. It is obvious that the temperature at the key point is unaffected by the degree of prevailing temperature inversion, since it is the point from which the temperature inversions are measured and the one point where the degree of temperature inversion is always zero. Thus the value of the factor $f(I)$ can be made to vary more or less at will merely by moving from one point to another, and can be made to vanish at the key point.

The general equation for minimum temperature under ideal frost conditions at the key station can now be written:

$$y = f(d) + f(h). \quad (3)$$

FACTORS DIFFICULT OF MATHEMATICAL EXPRESSION

The ideal frost conditions occur but rarely in nature. Other factors are required if Equation 3 is to be made applicable to all nights.

1. *Effects due to changing weather conditions.*—If warmer or colder air were brought in from surrounding regions by the general atmospheric circulation, or if the absolute humidity were changed by this wind movement, it is obvious that the minimum temperature would be affected.

2. *Effects due to mechanical action of night winds.*—Such winds are of frequent occurrence. They disturb the air drainage, even to the extent of shifting the key point; raise the surface temperature by mixing the warmer air aloft with the colder air near the ground (**13**); and sometimes completely prevent the inversion of temperature.

3. *Effects due to cloudiness.*—Clouds of any type composed of water droplets interfere with free radiation, but dense lower clouds have greater effect than any other kind (**14**) (**15**). High thin clouds, composed of ice crystals, have little or no effect on the rate of cooling through radiation.

Any one of the factors just named could have sufficient influence by itself to prevent, to a large extent, the usual nocturnal fall in temperature, by disrupting the normal processes under which the surface temperature is made to fall. Many nights which would prove damagingly cold under ideal frost conditions are characterized by temperatures well above the danger point, due to the action of winds. Clouds overspreading the sky in the latter portion of the night have been responsible for an abatement of threatened frost on many occasions, and fog often prevents it. Changes in absolute humidity during the night sometimes alter the rate of free radiation and cause minimum temperatures other than would have been experienced without such change.

These factors are difficult of mathematical expression. Their effects can be approximated from a study of current weather maps, and esoterically interpreted in terms of the number of degrees by which the free radiation minimum temperature formula estimate must be modified. Minimum temperature formulas, no matter how accurate they may be on nights when the ideal frost condition prevails, can not be used indiscriminately; nor has it been found possible to construct minimum temperature formulas of any practical value for all nights. The best solution to the problem of minimum temperature forecasting is to express mathematically the factors that can be so expressed and approximate the others. The minimum temperature formula is thus a means to an end and not an end in itself.

SELECTION OF DATA

Theoretically only the temperature data obtained on radiation nights should be used in the construction of the minimum temperature formula. In actual practice, however, it is found that there are very few nights during the frost season when ideal radiation conditions obtain. On many nights when frost gives real concern there is more or less wind or cloudiness or both over a portion of the night. But the fact remains that on most nights when frost does occur, ideal radiation conditions are present during the greater portion of the night, and under these conditions the minimum temperature closely approximates that which would have been experienced under ideal frost conditions.

It would appear that when the normal fall in temperature is interrupted by adverse radiation conditions of a local nature, such as intermittent breezes or occasional cloudiness, with a return to the ideal conditions after a short interval, the fall in temperature is accelerated until the effects of the temporary temperature rise are obliterated and the normal fall is again resumed. There is no ready way of explaining this phenomenon except by considering that the nonradiation conditions are intensely local during any given interval and that the great system of air drainage is not permanently affected to any material extent.

Data from which minimum temperature formulas are to be derived should include all nights when the minima at the key station were 32° F. or lower, with the occasional rejection of data obtained on cold nights when radiation conditions did not preponderate, and some additional data taken under ideal frost conditions even though the key station minima were above freezing.

POINTS OF SIMILARITY IN FORMULAS

It is a point in common with all minimum temperature formulas that after the factors to be correlated have been selected, the actual construction of the formula is patterned after the same general method. A "dot chart" is prepared by plotting one factor against another. The problem is then concerned with a determination of the line of "best fit" (**17**) to the data on the dot chart. The finished formula itself is simply the mathematical expression of this line.

The construction of the formula from data accumulated over a number of years presupposes an average condition in the moisture content of the surface soil. That the extreme condition in soil moisture is an important factor in minimum temperature forecasting is quite evident from the practical application of the hygrometric formula during periods of extreme drought or immediately after periods of heavy rainfall. When the surface soil is abnormally dry, minimum temperatures lower by from 1° F. to 3° F. than the hygrometric formula estimate usually are experienced. When the surface soil is thoroughly soaked with rain, the minimum temperature experienced usually is from 1° F. to 3° F. higher than the indicated minimum temperature by formula. By judicious use of this principle it often is possible to improve upon the accuracy of the formula estimate.

TEMPERATURE FORMULAS SUGGESTED

Many investigators have suggested from time to time empirical formulas designed to evaluate the minimum temperature from factors which can be assigned definite values in the late afternoon or early evening. These formulas can be placed, roughly, into three mathematical groups:

Group 1. $y=f\ (T)$.
Group 2. $y=f\ (d)$.
Group 3. $y=f\ (d)+f\ (h)$.

It is now proposed to deal with each group by itself, first listing by numbered paragraphs the various formulas as they have been proposed by their originators, and later critically discussing each formula in a paragraph numbered to correspond. The following mathematical conventions will be used throughout the remainder of the paper:

y is the minimum temperature,
d is the temperature of the dew point at an afternoon observation,
h is the relative humidity coincident with d,
n is a number deduced from study of data,
V_d is a variable depending on d, and,
V_h is a variable depending on h.

FORMULAS IN GROUP 1

1. "Median-hour" relationship suggested by Beals (**18**). The average time of occurrence of the temperature halfway between the maximum and minimum temperatures is found. The temperature reading taken at the time of the median is then subtracted from the maximum temperature and the remainder is used to indicate the approximate fall that will occur between the median and minimum temperatures.

2. "Post-median-hour" relationship suggested by Thomas (**19**). The number of degrees in temperature between the maximum and the temperature registered at 10 p. m. is considered to be two-thirds of the fall between maximum and minimum.

3. "Pre-median-hour" method originated by Alter (**21**). The trend of temperature fall in the early evening is used by the forecaster to predict the median-hour temperature by extrapolation, and thus to arrive at an earlier approximation of minimum temperature by the true median-hour method.

4. "Maximum-minimum" relationship proposed by Nichols (**22**) (**23**). The minimum temperature is considered to be a direct function of the preceding maximum temperature, so that when the maximum temperature is known, the minimum temperature can be determined.

5. "Daily temperature range" method formulated by Smith (**26**). The average, greatest, and least daily temperature range is computed by semimonthly periods, and the values used in minimum temperature forecasting after the maximum temperature is known.

DISCUSSION OF FORMULAS IN GROUP 1

1. The median-hour formula is based upon the general principle of assumed harmonic relationship between time and temperature under ideal frost conditions. Unless a special development of the principle is made, however, there is one type of night that occurs rather frequently under ideal frost conditions wherein the median-hour

formula would not apply. This type of night occurs when the dew-point temperature is reached, or closely approached, near the hour of the median. How long the air temperature will then remain nearly stationary, or continue to fall at a constantly decreasing rate, or whether the air temperature will fall lower than the dew point at all, are matters not related to the rate of effective radiation preceding the median hour. Since there is no evidence of any such special development having been made, it is probable that some of the inaccuracies in the application of the median-hour formula are due to this necessity for segregation of data.

Many investigators who have tried to utilize this type of formula have reported unsatisfactory results. It would appear that the median-hour formula is open to some serious practical objections.

To give satisfactory results any type of minimum-temperature formula must have some application on nights when ideal frost conditions do not obtain at all times between sunset and sunrise. Many cold nights are preceded by cloudy afternoons, as frost is observed frequently following the passage of a cyclone, and the time of occurrence of the maximum temperature is affected to such an extent as to impair the application of the median-hour method.

A rapid drop in air temperature in the early part of the night is quite often followed, in many districts, by local winds which cause fluctuating temperatures over short intervals. In fact, in some places, the topography is such that the fall in temperature must be considered as a causative agent in the production of local winds, as, for example, the well-known phenomenon of mountain and valley winds. The median-hour method stakes all on the air temperature at a certain instant, yet the temperature at this instant is often affected by local conditions.

The time of occurrence of the median hour in many sections of the country is so late that it is impracticable to use the formula in the preparation of forecasts.

Changes in absolute humidity near the time of the median will seriously impair the application of this method by changing the rate of effective radiation after the median temperature occurs. Such absolute humidity changes are frequently observed in mountainous country due to reversal of winds aloft under special conditions.

2. The post-median-hour relationship is open to the same objections as those just listed for the median-hour formula, except that the lateness of the zero hour detracts even more from its practical use.

3. The pre-median-hour method also is subject to the same criticisms. The method was originally devised to overcome one of the principal objections to the median-hour formula, namely, the delay in the preparation of the minimum temperature forecast while waiting for the median hour to be reached. It is evident that the chances for error in the approximation of the median-hour temperature would make the forecasts by this method subject to more risk than by the median-hour method itself, without any hope of attaining greater accuracy in the predictions.

4. The maximum-minimum temperature formula simply states that with a given maximum a certain range will ensue and a given minimum be reached. When it is considered that with any given maximum temperature a variety of values for absolute humidity are observed in practice, it is evident that differences in the rates of free radiation of the earth's heat will occur during different nights and consequently a variety of minimum temperatures are experienced. These formulas, then, appear to be faulty at their source.

Nichols, who is the chief proponent of the maximum-minimum type of formula, in the endeavor to improve the usefulness of the basic relationship, has extended the application by a complex system of type classification wherein five classes of weather conditions are recognized (**24**). He concludes, however, that the relationship is inferior to the hygrometric correlation and that "* * * greater inaccuracy is likely to result from incorrect classification than from inaccuracies in the formulas when correctly applied" (**25**).

5. The daily temperature range method is a variation of the maximum-minimum method previously discussed and is open to the same objections.

From a purely abstract consideration of values inherent in temperature formulas of the form $y=f(T)$ it would seem that in so far as accuracy is concerned they would range in the following order:

1. Postmedian-hour formulas.
2. Median-hour formulas.
3. Premedian-hour formulas.
4. Maximum-minimum formulas.

The postmedian-hour formulas would be the most accurate and the maximum-minimum formulas the least accurate, owing to the variation in the time to elapse before the occurrence of the minimum temperature after the determination of T. A forecast prepared one or two hours in advance of an event certainly should be expected to have greater accuracy than a forecast prepared 12 to 14 hours in advance.

In actual practice, however, the minimum temperature forecast must be made and disseminated long before the time of occurrence of the median hour in most parts of the country. This fact alone excludes from serious consideration all the formulas of this group except the maximum-minimum and certain of the premedian-hour formulas. Thus, the only formulas of the form $y=f(T)$ available for practical use are the least accurate of the group.

FORMULAS IN GROUP 2

1. "Evening dew-point" relationship proposed by Humphreys (**3**): The temperature is assumed not to fall below the value of the coincident dew-point temperature, which, for forecasting purposes, is considered to be closely the same as the evening dew point. In other words, the minimum temperature is determined by the value of the evening dew point.

2. "Wet-bulb" method originated by Ångström (**27**): A constant is subtracted from the wet-bulb temperature at sunset and the remainder indicates the ensuing minimum temperature.

3. "Wet-bulb-minimum temperature" method proposed by Keyser (**28**): The average difference between the wet-bulb temperature at 5:00 p. m., and the ensuing minimum temperature is found. The 5:00 p. m. wet-bulb temperature, when decreased by the amount of average difference, is the estimated minimum temperature.

4. "Depression of the evening dew-point" method tried by Smith (**29**): The difference obtained by subtracting the evening dew-point temperature from the coincident air temperature at an evening observation is used in correlation with the difference obtained by subtracting the evening dew-point temperature from the ensuing minimum temperature. The relationship thus determined enables the minimum temperature to be computed from evening observational data.

5. "Depression of the dew point below maximum temperature," method originated by Nichols (**31**): The difference obtained by subtracting the temperature of the evening dew point from the preceding maximum temperature is argued against the ensuing range in temperature between maximum and minimum. The relationship determined enables the minimum temperature to be computed from the evening observational data.

DISCUSSION OF FORMULAS IN GROUP 2

1. Soon after its initial formal statement, Smith (**33**) provided a refutation based on observational fact of the principle inferred in the evening dew-point formula, namely that the minimum temperature would not be lower than the temperature of the evening dew point. Other investigators (**11**) and the meteorological records taken in connection with fruit-frost work establish the soundness of Smith's argument. Generally speaking, the relationship can not be consistently demonstrated except for elevated stations, well placed in the inversion layer. At key stations the minimum temperature often is lower than the evening dew point by more than 8° to 10° F., with occasional extremes of more than 20° F. The formula, therefore, is inherently faulty.

2. The wet-bulb temperature is determined when the coincident dry-bulb temperature and the dew point are known. Under any given condition its value is greatest when the relative humidity is 100 per cent, for it is then that the wet and dry bulb temperatures coincide, and its value becomes progressively less with decreasing relative humidity. In other words, wet-bulb temperatures and relative humidity change in direct ratio with unchanging dew point.

Assume the dew point to remain constant. The wet-bulb temperature can now be made to vary by causing that of the dry bulb to change. If the difference between the minimum temperature and the wet-bulb readings at an evening observation is always to be a constant, then we are justified in assuming that the difference between the minimum and the dew point, which remains fixed, will likewise be a constant *for all values of relative humidity.*

But when the dew point is fixed, Equation 3 assures us that the variation of the minimum temperature from the evening dew point depends upon the relative humidity, or,

$$y-d=f(h)$$

and investigators are universally agreed that the right-hand member of this equation is not a constant. The hygrometric dot charts that comprise Supplement 16, Monthly Weather Review, offer ample proof of this contention.

During the winter of 1922–23, Dague experimented with Ångström's formula in a district in southern California and found in this season that the variation of the minimum temperature from the evening wet-bulb temperature was not a constant, but had values ranging from +12 F. to +23° F. (**35**).

We conclude that Ångström's wet-bulb formula is fundamentally in error and must be rejected.

3. This formula is of the same order as the one previously discussed. Keyser, the proposer himself, admits its inferiority (**28**).

4. Some rather unsatisfactory attempts have been made to correlate the depression of the evening dew point with the variation of the minimum temperature from the evening dew point (**29**). The difficulty with the relationship here expressed lies in the fact that when the depression of the evening dew point is computed a pure number is obtained which may be the same for widely differing values of absolute humidity or air temperature. Thus, for example, the number 21, which here is taken to represent the difference between the air temperature and dew-point temperature at an evening observation, may occur under any condition of absolute humidity, as the air temperature may vary in such manner that a differential of 21 always is maintained; or the same number 21 may be obtained with any value of air temperature, depending on the absolute humidity. The appearance of the differential 21, therefore, is no indication whatever of the amount of moisture in the air. Many widely differing rates of free radiation, consequently with widely differing minimum temperatures, can occur with any designated differential.

5. The depression of the evening dew-point temperature below the maximum temperature formula simply states that a certain range in temperature will ensue following the occurrence of a certain differential between maximum temperature and evening dew point. But the temperature difference between afternoon maximum and evening dew point is not a measure of the absolute humidity, for these latter are independent factors and almost any difference can obtain between them when one or the other is regarded as being fixed. With any designated differential between afternoon maximum temperature and evening dewpoint, therefore, a variety of values for absolute humidity may exist, and with them, a variety of nocturnal temperature ranges instead of but one.

FORMULAS IN GROUP 3

Strictly speaking, there has been but one fundamental relationship of the form $y=f(d)+f(h)$ set forth by investigators, although at first glance several different formulas seem to appear. The difference lies not in the basic relationship itself but in the methods used to express this relationship mathematically. The difference in formulas, therefore, is one of form rather than of concept.

1. "Hygrometric" method. The fundamental concept underlying all hygrometric formulas in this group is that the ensuing minimum temperature will be greater or less than the evening dew-point temperature by an amount depending on the relative humidity. In this method the difference obtained by subtracting the evening dew-point temperature from the minimum temperature is argued against the evening relative humidity. The relationship is used to predict the minimum temperature from evening observational data.

DISCUSSION OF FORMULAS IN GROUP 3

1. The hygrometric relationship is the only one so far proposed that conforms strictly to equation 3, evolved for predicting the minimum temperature at the key station under ideal frost conditions. It is to be expected that the hygrometric formulas should be the most satisfactory. The greater part of the published literature on minimum temperature formulas has to deal with those based on this relationship, and the consensus of investigators is almost unaminous in awarding to them the wreath of superiority. These formulas, then, deserve more than a passing glance.

The hygrometric relationship was first proposed by Donnel in 1910, while working on Boise, Idaho, frost records (36) (20) (37). Donnel developed a hygrometric formula for Boise based on psychrometric observations taken at 6:00 p. m. through the assumption that the line of best fit to the hygrometric dot chart was a straight line whose equation is of the form

$$y=d-\frac{h-n_1}{n_2}.$$

So far as known, no extended practical use in forecasting was ever made of this equation and it was regarded as unsatisfactory except for a certain narrow range of relative humidity and dew point, where fairly consistent results were obtained.

In August, 1917, Smith published in the Monthly Weather Review the results of an investigation of the hygrometric relationship based on fruit-frost work in Ohio since 1915 (34). He followed a different line of attack than Donnel and determined from the correlation coefficient the probable existence of a linear relationship, later calculating the straight-line formula by the method of least squares (38) and expressing the hygrometric formula in the following convention:

$$Y=a-bR$$

where

Y is the difference between the minimum temperature and the evening dew point, or $Y=y-d$;

R the evening relative humidity, or $R=h$; and

a and b are two numbers deduced from study of data, or

$a=n_1/n_2$ and $b=1/n_2$.

Marvin has already demonstrated the mathematical identity of these original Smith and Donnel equations (36). In this case both investigators arrived at exactly the same place by following different routes; one of which was considerably longer and more difficult than the other.

In the spring of 1917, Young, while engaged on fruit-frost work at Medford, Oreg., investigated the hygrometric formula proposed by Donnel for Boise (39). He used data taken at 5:00 p. m. instead of 6:00 p. m., and found that the original Donnel equation gave fairly accurate results at Medford on cold nights when the sky was cloudy at observation, but indicated values too high when the weather at observation was partly cloudy or clear. This defect was remedied by writing separate equations for these two classes of nights. In effect, this amounted to subtracting 2½° F. and 5° F., respectively, from the original formula to make it applicable on partly cloudy and clear nights.

In using these equations it was found that when the 5:00 p. m. dew point was below 30° F. or above 40° F., the minimum temperature indicated by the formula consistently varied from the actual minimum temperature by an amount that was nearly constant with the same dew point. Also, when the relative humidity at 5:00 p. m. was above 67 per cent the formula needed revision upward by an amount depending on the relative humidity. Thus was developed the device called the "method of arbitrary corrections," by means of which the basic straight-line formula was made to take on an irregular *curvilinear* form. Later, the results of this investigation were published in Supplement 16 (16). Young expressed his formula after this form:

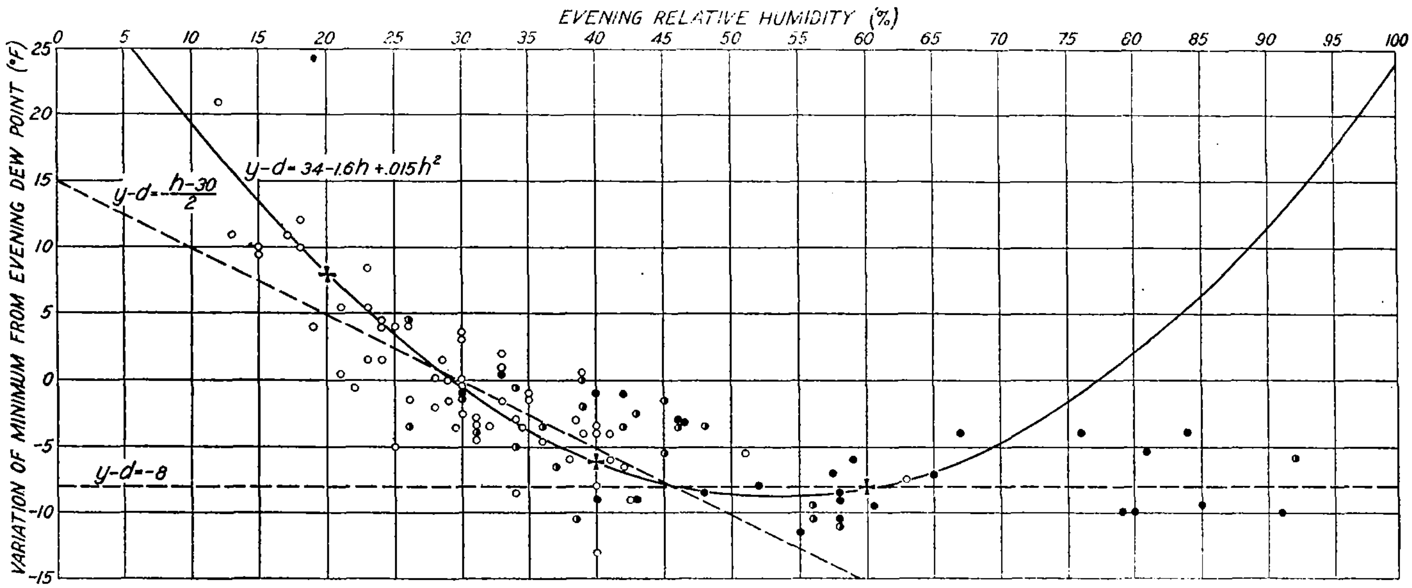


FIGURE 1.—Hygrometric dot chart for key station at Medford, Oreg., based on spring frost records from 1917 to 1928. State of weather at observation is shown by symbols. The parabolic curve of formula 1 and the straight lines of formula 3 have been plotted on the chart

$$y=d-\frac{h-n}{4}+V_d+V_h.$$

He, therefore, was the first to use in actual minimum temperature forecasting a hydrometric formula which expressed a curved line of best fit to the data from which it was constructed.

In July, 1919, Smith published in Supplement 16 a method for fitting parabolic curves to hygrometric dot charts after the Marvin "star-point" system (17) and the mathematical expression of hygrometric formulas in the form of parabolic equations:

$$y-d=n_1+n_2h+n_3h^2.$$

Nichols found that a rectilinear hyperbola would, in some cases, give better correlation than the parabolic type curve (32).

Nichols (25) and Keyser (40) brought forward the idea that dot-chart data in all cases might not best be represented by any of the simpler mathematical curves and suggested the extension of the "star-point" system to the production of a curve drawn by eye alone. This free-hand curve could than be used directly from the dot chart without recourse to further mathematical methods.

Since these various hygrometric formulas and forecasting methods are all based on the fundamental relationship expressed in Equation 3, it is impossible to make selection of the best one of the group without some investigation into the relative merits of all. Perhaps the best method of making this comparison would be to develop all the formulas from the same data, and then to determine the *relative* worth by applying the fromulas back on the data from which they were derived. If one formula were superior to another this method surely would bring it to attention.

An intensive study of hygrometric data taken at the Medford, Oreg., key station during the spring frost seasons 1917 to 1928, inclusive, is now proposed. In this study only the finished dot charts, formulas, and frequency diagrams of the investigation will be presented.

Reference is made to the hygrometric dot chart shown in Figure 1 from which a number of formulas are now to be derived. On this chart are plotted data secured on 106 nights at the Medford, Oreg., key station during the spring seasons over a 12-year interval. The data have been selected to include all nights when the minimum temperature fell to 34° F. or lower, provided the sky was clear over the greater portion of the night. Symbols on the chart show the state of the sky at the time of observation. Naturally, a large portion of the data were taken on nights not ideal for free radiation and the wide scattering of the dots may be attributed to this reason. The rejection of data on nights when the sky did not clear until after midnight would result in a much more compact arrangement of the dots. This was not done because it was desired to deal with the situation under conditions as they occur in actual practice.

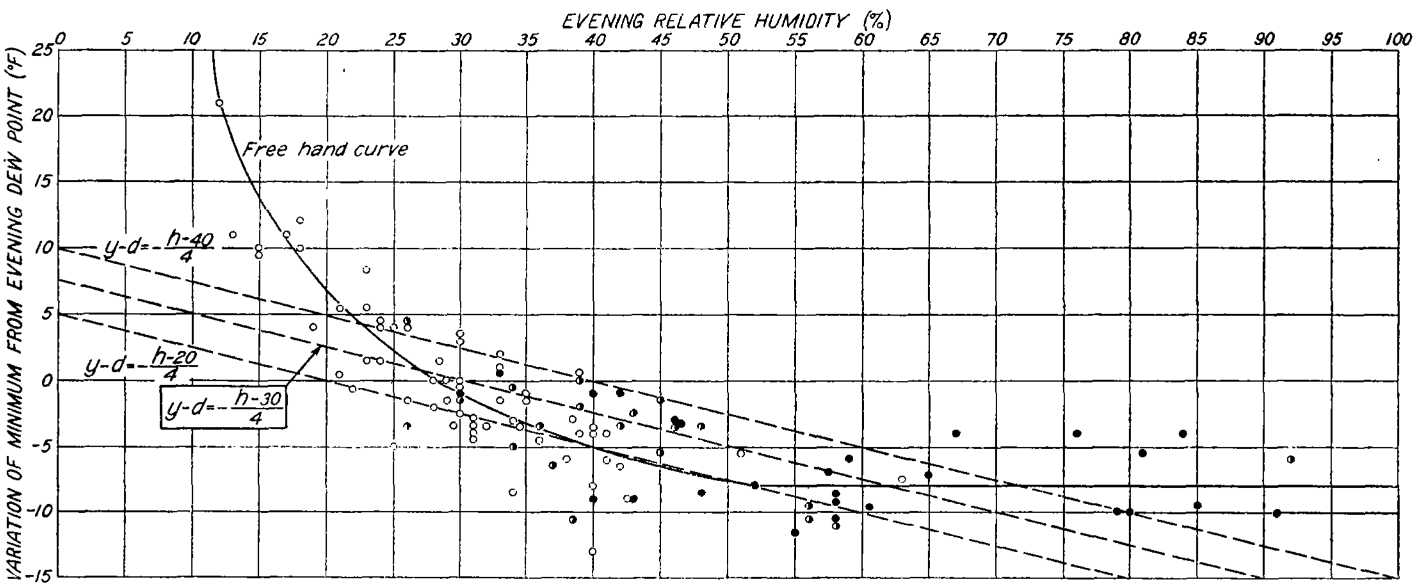


Figure 2.—Hygrometric dot chart for key station at Medford, Oreg., based on spring frost records from 1917 to 1928. State of weather at observation is shown by symbols. The free-hand curve of formula 5 and the basic straight lines of formula 7 have been plotted on the chart

SMITH FORMULA

The first formula to be considered is the parabolic type patterned after the Smith method. Three "Star points" have been selected as indicated on Figure 1; one point for each 35 dots. The hygrometric formula is expressed:

$$y-d=34-1.6h+.015h^2. \quad \text{Formula 1.}$$

and its position shown on the dot chart. (See figure 1.) When this formula is applied back on the data from which it was derived the results shown in the frequency diagram are obtained. (See figure 3.)

If, now, the method of arbitrary corrections be applied to the Smith formula as a base, the new formula becomes

$$y-d=34-1.6h+.015h^2+V_d+V_h. \quad \text{Formula 2.}$$

with values for the variables shown in the table below:

d	V_d	h	V_h
7° to 24°	+2	12% to 21%	−2
26° and 27°	+1½	22% to 25%	−2
28° to 30°	0	26% to 31%	−1
31°	+1½	32% to 34%	+½
32° and 33°	+1	35% to 39%	+1½
34°	0	40% to 42%	+1
35° and 36°	−1½	43% to 51%	+3½
37° and 38°	−½	52% to 59%	0
39° and 40°	−1½	60% to 76%	−½
41° to 44°	−3	77% to 85%	−11½
		86% to 92%	−21

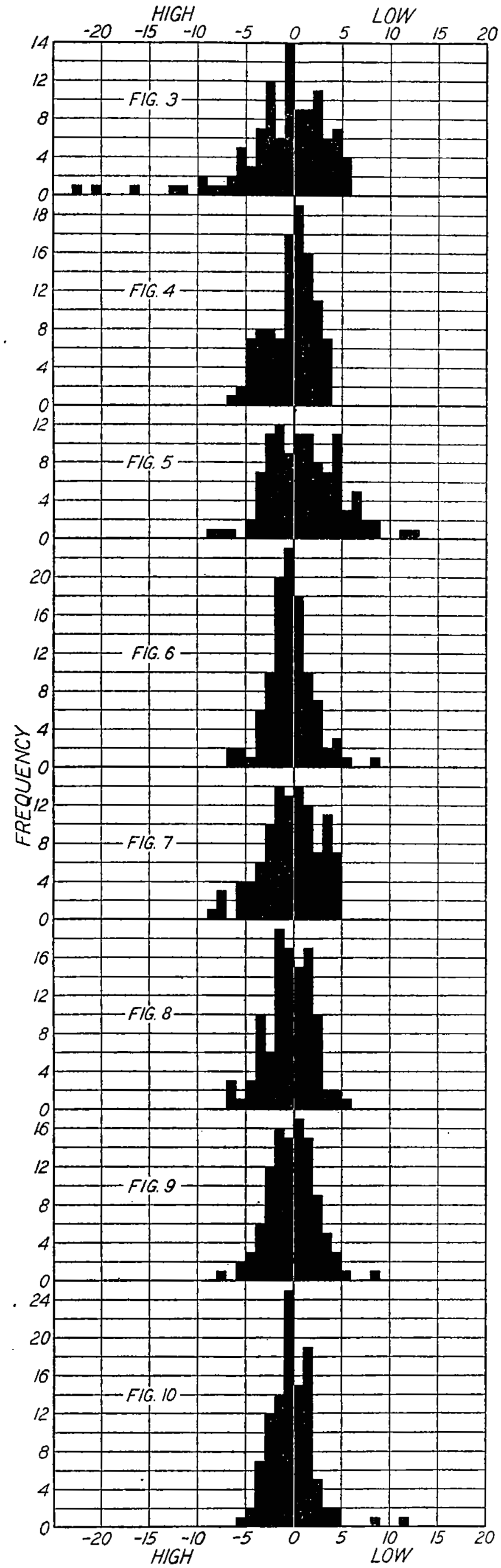
When Formula 2 is applied back to the original data the results shown in the frequency diagram in Figure 4 are obtained.

It is obvious from the frequency diagrams that Formula 2 gives much better application to the data than Formula 1. Not only is the amplitude of variation greatly reduced, but also the accuracy is materially increased.

DONNEL FORMULA

Examination of the dot chart in Figure 1 shows at once that it would not be possible to select a single straight line to represent the data faithfully. Accordingly a subdivision of the data has been made in order to apply the Donnel method.

State of weather at observation	Relative humidity at observation	Formula 3
Clear	44% or less	$y=d-\frac{h-30}{2}$
Partly cloudy or cloudy.	Any value	$y=d-8$
Clear	45% or more	$y=d-8$



FIGURES 3-10.—(3) Frequency diagram showing the application of the Smith parabola, formula 1, to the data from which it was derived. (4) Frequency diagram showing the application of the Smith parabola after being modified by the method of arbitrary corrections, formula 2, to the data from which it was derived. (5) Frequency diagram showing the application of the Donnel straight-line formula, formula 3, to the data from which it was derived. (6) Frequency diagram showing the application of the Donnel straight-line formula after being modified by the method of arbitrary corrections, formula 4, to the data from which it was derived. (7) Frequency diagram showing the application of the Nichols free-hand curve formula 5, to the data from which it was derived. (8) Frequency diagram showing the application of the Nichols free-hand curve after being modified by the method of arbitrary corrections, formula 6, to the data from which it was derived. (9) Frequency diagram showing the application of the Young formula, formula 7, to the data from which it was derived. (10) Frequency diagram showing the application of the Young formula after being written in such form as to give greater weight to peculiar local conditions, formula 8, to the data from which it was derived

The position of these lines is shown in Figure 1, and the application of the formula is presented in the frequency diagram in Figure 5.

Applying the method of arbitrary corrections to Formula 3 changes it to this form:

State of weather at observation	Relative humidity at observation	Formula 4
Clear	44% or less	$y=d-\frac{h-30}{2}+V_d+V_h$
Partly cloudy or cloudy.	Any value	$y=d-8+V_{d1}+V_{h1}$.
Clear	45% or higher	(same)

where the variables take on the following values:

d	V_d
7° to 24°	+4½
26° to 29°	+1½
30° to 35°	0
36° to 43°	−3

d	V_{d1}
26° to 31°	+6½
32° to 35°	+4
36° to 38°	+1
39° to 44°	−1

h	V_h
12% to 30%	−1
31% to 34%	0
35% to 44%	+1½

h	V_{h1}
26% to 37%	+1
38% to 92%	0

The application of Formula 4 is shown in Figure 6. Again has the amplitude of variation been reduced and the accuracy and dependability of the formula greatly increased.

NICHOL'S FREE-HAND CURVE FORMULA

From the dot chart shown in Figure 2 an attempt has been made to draw a free-hand curve of best fit to the data. This curve has been plotted in Figure 2 and can be expressed as a mathematical formula in this manner:

$$y=d+V_h. \qquad \text{Formula 5.}$$

where the variable takes on fixed values when the relative humidity has been determined:

h	V_h	h	V_h
12%	+21	33%	−2½
13%	+18	34%	−3
14%	+16	35%	−3
15%	+14	36%	−3½
16%	+12½	37%	−4
17%	+11	38%	−4
18%	+9½	39%	−4½
19%	+8	40%	−5
20%	+7	41%	−5
21%	+6	42%	−5½
22%	+5	43%	−6
23%	+4	44%	−6
24%	+3	45%	−6
25%	+2½	46%	−6½
26%	+1½	47%	−7
27%	+1	48%	−7
28%	0	49%	−7
29%	−½	50%	−7½
30%	−1	51%	−7½
31%	−1½	52% to 100%	−8
32%	−2		

The free hand curve closely resembles the Smith parabola, Formula 1, for relative humidities between 10 per cent and 52 per cent, but becomes the same as the Donnel formula, Formula 3, for relative humidities above 52 per cent. It is in reality a combination of the better parts of both Formula 1 and Formula 3 and the frequency diagram showing its application assures us that it is better than either the Smith of the Donnel formulas taken separately. See Figure 7.

The addition of the method of arbitrary corrections to Formula 5 changes it to this form:

$$y=d+V_d+V_h. \qquad \text{Formula 6.}$$

where the variables take on values as follows:

h	V_h
12%	+18½
13%	+15½
14%	+13½
15%	+11½
16%	+10
17%	+8½
18%	+7
19%	+5½
20%	+4½
21%	+3½
22%	+4
23%	+3
24%	+2
25%	+1½
26%	+½
27%	0
28%	−1
29%	−1½
30%	−1½
31%	−2
32%	−2
33%	−2½
34%	−3
35%	−3
36%	−3½
37%	−4
38%	−4
39%	−4½
40%	−5
41%	−5
42%	−5½

h	V_h
43%	−6
44%	−6
45%	−6
46%	−6½
47%	−7
48%	−7
49%	−7
50%	−7½
51%	−7½
52% to 100%	−7

d	V_d
7° to 24°	+ ½
25° to 27°	+1
28° to 30°	0
31° to 33°	+2
34°	0
35° and 36°	−1
37° and 38°	− ½
39° and 40°	−1
41° to 44°	−2
7° to 24°	+ ½
25° to 27°	+1
28° to 30°	0
31° to 33°	+2
34°	0
35° and 36°	−1
37° and 38°	− ½
39° and 40°	−1
41° to 44°	−2

The application of Formula 6 is shown in Figure 8. As before, an improvement of the original formula has been effected by these simple means.

YOUNG FORMULA

Young uses a base equation of the Donnel type for each of three classes of nights, depending on the state of weather at observation. The method of arbitrary corrections is applied to the formula as a whole. The base equations in this case are plotted in Figure 2 and the hygrometric formula expressed:

State of weather at observation	Formula 7
Clear	$y=d-\frac{h-20}{4}+V_d+V_h.$
Partly cloudy	$y=d-\frac{h-30}{4}+V_d+V_h.$
Cloudy	$y=d-\frac{h-40}{4}+V_d+V_h.$

d	V_d	h	V_h
7° to 24°	+9	12% to 21%	+1
25° to 30°	+2	22% to 25%	+½
31° to 34°	+1½	26% to 39%	−½
35° to 38°	−1½	40% to 59%	−1
39° and 40°	−3	60% to 79%	+2
41° to 44°	−4½	80% to 92%	+5½

The application of this formula is shown in Figure 9. It is a fact worth noting that the amplitude of variation in Figure 9, roughly from 5° too high to 5° too low, is nearly the same as shown in Figures 4, 6, and 8. The types of frequency curves in these figures, to, are not, entirely dissimilar. If the mass of data were greater the similarity undoubtedly would be more pronounced.

Assuming that the data on the dot chart have but one correct interpretation, only one line of best fit can be drawn. The evidence in this case points strongly to the idea that the line of best fit is irregular, since the addition of the arbitrary corrections to any base formula results in an irregular line unless the base formula itself is the line of best fit, in which case the corrections vanish. In each case here considered the irregular line formula produced by the corrections gives better application to the data than the regular base formula by itself. And in each case the irregular line formula when applied to the original data produces the same general type of frequency curve, regardless of the type of base formula used in the derivation. We conclude, therefore, that the addition of the method of arbitrary corrections to *any* base line or curve will produce the same irregular line of best fit to any hygrometric dot chart.

We are now in position to select the best type of hygrometric formula. Since the formulas are identical, we may select any in which the method of arbitrary corrections has been considered, and since the base formula can be any line or curve, we are justified in choosing the simplest, which is, of course, the straight line. We make selection, then, of the formula which uses a straight line as a base but changes to an irregular curve, if necessary, by the addition of the arbitrary corrections. These specifications fit the formula constructed by Young's method.

In seeking out the hygrometric formula to apply to different key stations, however, the investigator must always be on the alert to recognize any special conditions with which he may be confronted. While a formula developed after the methods just outlined will give good application at any well chosen key station, a different construction placed on the data may result in an even better formula. For instance, Formula 7 can be rewritten in such form as to give greater weight to local conditions of cloudiness.

State of weather at observation	Formula 8
Clear	$y=d-\frac{h-20}{4}+V_{d1}+V_{h1}.$
Partly cloudy	$y=d-\frac{h-30}{4}+V_{d2}+V_{h2}.$
Cloudy	$y=d-\frac{h-40}{4}+V_{d3}+V_{h3}.$

d	V_{d1}	h	V_{h1}
7° to 24°	+10	12% to 21%	0
25° to 29°	+ 3½	22% to 26%	− ½
30° to 35°	+ 2	27% to 30%	−1
36° to 43°	− 2	31% to 39%	+ ½
		40% to 51%	−1

d	V_{d2}	h	V_{h2}
26° to 31°	+ ½	26% to 38%	−2½
32° to 42°	− 2	39% to 63%	+1

d	V_{d3}	h	V_{h3}
26° to 37°	+ 1	30% to 52%	−3
37° to 39°	− 1½	53% to 65%	− ½
40° to 44°	− 4	66% to 91%	+4

The frequency diagram in Figure 10 giving the application of this formula shows that the amplitude of variation has been reduced to 3° for 103 of the 106 cases of frost used to construct the formula.

METHOD OF CONSTRUCTING YOUNG FORMULAS

Since it is a conclusion of this study that the Young type of formula is superior to the others, it may be well to state briefly, step by step, an easy method of developing this type of formula.

1. Construct a dot chart after the methods outlined by Smith (**30**). This is the initial step in the construction of any minimum temperature formula. Either make a separate chart for clear, partly cloudy, and cloudy nights, or better, place all data on one chart, denoting segregation by use of inks of different color. A small index number keyed to the original data sheet is placed near each dot for identification purposes.

2. Draw a straight line on the chart to represent, as nearly as a straight line may, the data. To simplify the formula, care should be taken to select a line capable of whole number expression, even though the position of the true line is shifted slightly. Draw a line for each class of data if one line will not represent the whole.

3. Express this line in the form:

$$y=d-\frac{h-n_1}{n_2}$$

where n_1 is the number measured on the horizontal relative humidity scale where the line intersects the X-axis, or zero line of departure, and, n_2 is the number of units measured by projecting the line on the horizontal relative humidity scale, that the line changes in order to pass through one unit of variation by projection on the vertical scale of departures.

4. To facilitate the calculation of the arbitrary corrections, the data for each day should be written on a card. The example shown below indicates the data to be considered and the method of preparing the cards. The headings, of course, can be omitted.

Index No.	Evening dew point	Evening relative humidity	Minimum temperature	State of weather
116	30. 5°	76%	28. 4	Cloudy. Cleared 9:30 p. m.

5. Using the straight line formula just calculated, go through the cards and calculate the formula minimum temperature. Find the departure between this and the actual minimum temperature.

6. Now arrange the cards in the order of increasing relative humidity. At more or less regular intervals—say, about 10 cards—calculate the average departure of the formula minimum temperature estimate from the actual minimum temperature. This is the arbitrary correction to be applied to the formula over the range of relative humidity indicated. Apply this correction to the original formula estimate and calculate the new departure.

7. After the relative humidity corrections have been determined and applied, arrange the cards in the order of increasing dew point. Repeat the process just outlined, thus determining the arbitrary correction to be applied to the formula over the range of dew point indicated.

CONCLUSIONS

Under ideal frost conditions the air temperature at the surface falls during the night, due to the influence of several factors operating simultaneously. The cumulative effects of these several factors can be given mathematical expression, and the minimum temperature closely calculated from hygrometric data taken near sunset on the preceding day, provided care is taken to locate the point for which forecasts are to be prepared with due regard for local topography.

The forecast or key station should be placed as nearly as possible to the point where the cold air that drains from the surrounding slopes first gathers to form the nuceleus of the pool of cold air that covers the lowlands in the morning. By so locating the key station it is possible to avoid entirely the effects due to temperature inversion.

Minimum temperature forecasts by formula are not satisfactory for elevated stations. The errors introduced by the factor of temperature inversion are usually so great as to void any accuracy to be obtained by computation from the other factors. As a general rule it may be stated that the greater the influence of inversion on the minimum temperature at any station, the poorer will be the application of the minimum temperature formula. The formula must have at least as great a variation in application as the difference between the average and extremes of temperature departure due to inversion. Formula construction for an elevated city station is a hopeless task.

As the means of accomplishment are always less important than the ends to be attained, so is the hygrometric formula minimum temperature indications of less importance than the final forecast of minimum temperature, which properly is the result of processes both mathematical and empirical. But the fact remains that the hygrometric formula is an indispensible tool in the hands of the forecaster. Under any of the conditions where frost is formed a skillfully designed hygrometric formula based on key station records over at least a 10-year period offers a purely mathematical method of placing the minimum temperature estimate within 3° F., of the actual to be experienced fully 90 per cent of the time, and in most cases within 6° F., in the remaining 10 per cent. The skill of the forecaster is then directed toward an improvement of the original mathematical estimate within this narrow range.

The forecaster has many things to guide him. In general, it is easy to recognize the cases where the departrue will be in excess of 3° F. This occurs during serious freeze types of weather where the maximum temperature and dew point are unusually low, with an accompaniment of winds of decided force; or else when local winds and cloudiness are to be experienced during a portion of the night. The experience of the previous morning often is at hand. The amount of moisture in the surface soil sometimes offers a dependable means of correction. For the rest, the personal experience of the forecaster in his own particular district, and his personal ability to determine accurately the immediate weather from the current weather chart and translate these indications into the terms of degrees in temperature, is put to test.

The forecaster also must be able to differentiate between the conditions where frost will or will not occur. It is a fact that the majority of nights during the period when frost danger is most acute would be frosty if the sky remained clear and there was no surface wind. The hygrometric formula, being constructed from data taken on cold nights only, invariably indicates this tendency. But all nights during the frost season are not actually frosty. Some are cloudy; some are windy; and some are rainy. It is the forecaster's own job to segregate the nights when the formula will have application. Properly used, the formula will indicate about how low the temperature at the key station will fall under ideal frost conditions, and gives the forecaster a base upon which to build, but it has nothing but theoretical application on nights when frost is not experienced. The hygrometric formula is not constructed for use on nights like these

Of the many temperature formulas that have been suggested, some are inherently faulty and others have but limited use. The family of temperature formulas based upon the hygrometric relationship has been shown in this paper to be outstandingly superior. It so happens, however, that within this family of hygrometric formulas there is one that maintains a flexibility in construction and accuracy in practical application in a degree not attained by the others. This is the Young formula wherein the basic relationship is linear and close construction given to the data by a series of arbitrary corrections.

Much in useful accuracy is lost if the variables V_d and V_h are not computed. It is not enough to carry the formula development only to the point where the base formulas are determined, for although in such cases there is likely to be excellent correlation between the factors, values usually can be assigned the variables which will result in even closer correlation. The variables provide:

1. For the influence of local topography, and,
2. An empiric evaluation of $f\ (L)$.

If only a short record is available for a key station where it is desired to develop a formula, it will be found that not enough data are at hand from which to construct a satisfactory formula of the Young type. In this case the Smith or Nichols type of formula is recommended, as these formulas lend themselves readily to interpolation. Later, a formula after Young's method can be constructed.

Any of the other formulas in the hygrometric family, when given the same treatment of arbitrary correction as used in Young's formula, have the same flexibility and accuracy. The smooth mathematical line of best fit is altered by the corrections to approach the position of an irregular line of best fit. There is no evidence to support the idea that the data on the dot chart must conform to some standard mathematical line or curve. The evidence, rather, points to the contrary view that, in most cases, the line of best fit is an irregular curve. By the application of the method of arbitrary corrections all the formulas in the hygrometric group are reduced to the same general irregular form regardless of the form of the base formula, whether parabola, hyperbola, free-hand curve, or straight line. In fact, all hygrometric formulas wherein the arbitrary corrections have been considered can be shown to be mathematically identical.

Selection of the Young straight-line formula as the superior formula of the hygrometric group is made for the reason that, since all the formulas when given similar treatment by arbitrary corrections are mathematically identical, it is proper to select the simplest means of accomplishing a desired end.

LITERATURE CITED

(1) Air Drainage Explained. Charles F. Marvin. Monthly Weather Review. October, 1914. **42**: 583–585.
(2) Frost Protection. W. J. Humphreys. Monthly Weather Review. October, 1914. **42**: 562–569. Page 562.
(3) Same citation. Page 565.
(4) Influence of Exposure on Temperature Observations. Floyd D. Young. Monthly Weather Review. December, 1920. **48**: 709–711.
(5) Influence of Cover Crops on Orchard Temperatures. Floyd D. Young. Monthly Weather Review. October, 1922. **50**: 521–526.
(6) An Experiment to Determine the Effects of Cover Crops on the Frost Hazard. Eckley S. Ellison. Abstracted in Monthly Weather Review. October, 1922. **50**: 526.
(7) Further Study of the Relation between Cover Crops and Orchard Temperatures. Floyd D. Young. Monthly Weather Review. September, 1925. **53**: 387–391.
(8) Effect of Topography on Temperature Distribution in Southern California. Floyd D. Young. Monthly Weather Review. August, 1920. **48**: 462.
(9) Nocturnal Temperature Inversions in Oregon and California. Floyd D. Young. Monthly Weather Review. March, 1921. **49**: 138–148.
(10) Notes on the 1922 Freeze in Southern California. Floyd D. Young. Monthly Weather Review. November, 1923. **51**: 581–585.
(11) Temperature Survey of the Salt River Valley, Ariz. James H. Gordon. Monthly Weather Review. May, 1921. **49**: 271–274.
(12) Rate of Increase in Temperature with Altitude During Frosty Nights in Orange Groves in Southern California. Floyd D. Young. California Citrograph. March, 1920. Page 136.
(13) Frost Protection by Artificial Mixing of Air. Floyd D. Young. California Citrograph. February, 1924. Page 125.
(14) Smoke Cover and Direct Radiation in Frost Protection Floyd D. Young. California Citrograph. November, 1920. Page 6.
(15) Forecasting Minimum Temperatures in Oregon and California. Floyd D. Young. Monthly Weather Review, Supplement 16. Pages 53–60. Page 57.
(16) Same Citation. Pages 53–60.
(17) Introduction. Charles F. Marvin. Monthly Weather Review, Supplement 16. Page 5.
(18) Forecasting Frost in the North Pacific States. Edward A. Beals. Bulletin No. 41–W. B. No. 473. February 27, 1912. Page 39.
(19) Same Citation. Page 38.
(20) Same Citation. Page 17.
(21) Forecasting Minimum Temperatures in Utah. J. Cecil Alter. Monthly Weather Review, Supplement 16. Page 49.
(22) Predicting Minimum Temperatures in Grand Valley, Colo. Esek S. Nichols. Monthly Weather Review. May, 1918. **46**: 213–228.
(23) Notes on Damage to Fruit by Low Temperatures; Prediction of Minimum Temperatures. Esek S. Nichols. Monthly Weather Review, Supplement 16. Page 41.
(24) Same citation. Page 39.
(25) *Same citation.* Page 42.
(26) Frost Warnings and Orchard Heating in Ohio. J. Warren Smith. Monthly Weather Review. October, 1914. **42**:579.
(27) Studies of the Frost Problem, I. Anders Ångström. Geografiska Annaler. January, 1920, pages 20–32. Abstracted in Monthly Weather Review, November, 1920. **48**: 640–641 by J. Warren Smith.
(28) Damaging Temperatures and Their Calculation in Advance By Simple Arithmetic. Elgie M. Keyser. Proceedings of the Washington State Horticultural Association for 1922. Olympia, Wash. Pages 97–103.
(29) Predicting Minimum Temperatures from Hygrometric Data. J. Warren Smith. Monthly Weather Review, Supplement 16. Page 18.
(30) Same citation. Pages 6–19.
(31) Notes on Formulas for Use in Forecasting Minimum Temperatures. E. S. Nichols. Monthly Weather Review. December, 1926. **54**:499–501. Page 500.
(32) Same citation. Page 499.
(33) Predicting Minimum Temperatures. J. Warren Smith. Monthly Weather Review. August, 1917. **45**: 402–407. Page 402.
(34) Same citation. Pages 402–407.
(35) Report of Temperature Survey and Fruit-Frost Investigations, San Gabriel Valley District, Southern California. Charles I. Dague. Special report on file in Division of Agricultural Meteorology, Washington. Page 15.
(36) Historical Note. Charles F. Marvin. Monthly Weather Review, August, 1917. **45**:407.
(37) Report on Fruit-Frost Work in Southern Idaho. Edward L. Wells. Special report on file in Division of Agricultural Meteorology, Washington. June, 1910.
(38) Elementary Notes on Least Squares, The Theory of Statistics and Correlations, for Meteorology and Agriculture. Charles F. Marvin. Monthly Weather Review, October, 1916. **44**:551–569.
(39) Report on the Frost Warning Service in the Rogue River Valley, Oreg., for the Spring of 1917. Floyd D. Young. Special report on file in Division of Agricultural Meteorology, Washington, D. C. *Pages 14–24.*
(40) Calculating Temperature Extremes in Spokane County, Wash. E. M. Keyser. Monthly Weather Review, October, 1922. **50**:526–528. Page 527.